Appendix 1B

RADAR EQUATION & RADAR CROSS SECTION (RCS: 雷達截面積)

* Pozar, Microwave Engineering 4th Ed (Ch 14; Sec.14.3)

**Monostatic radar system**

Radia antenna gain: \( G \)  
Transmitting power: \( P_t \)

- **Transmitting (EM wave) power density** on the target:
  \[
  S_t = \frac{P_t G}{4\pi R^2} 
  \]
  *(more rigorous expression in EM equation: \( P_{av}(\theta, \phi) = P_{rad} G(\theta, \phi) \))

- **Scattered (or reflected) power density** (by the target) \( \Rightarrow \) very complicated !!

- **Define radar cross section (RCS) \( \sigma \):** \( \sigma(m^2) = P_s / S_t \) 雷達截面積

- **\( P_s \): total power scattered by the target** (in a given direction) ?

  \( \Rightarrow \) \( RCS \) depends on the target shape, size, wave incident- & reflection angles, wave polarization, ....

  \( \Rightarrow \) **Scattered (wave) power density** back to the radar (receiving) antenna is

  \[
  S_r = S_t \frac{\sigma}{4\pi R^2} = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} = \frac{P_t G}{(4\pi R^2)^2} \]

  **Receiving power** by the (radar) receiver

  \[
  P_r = S_r \frac{A_e}{(\text{receiving}) \text{ antenna effective area}} = \frac{P_t G \sigma}{(4\pi R^2)^2} \left( \frac{\lambda G}{4\pi} \right) = \frac{P_t G^2 \lambda \sigma}{(4\pi)^3 R^4} \propto R^{-4}
  \]

  **If the (radar) minimum detectable power** is \( P_{\text{min}} \)  
  the radar maximum detectable range \( R_{\text{max}} \)

  \[
  P_{\text{min}} = \frac{P_t G^2 \lambda \sigma}{(4\pi)^3 R_{\text{max}}^4} \Rightarrow R_{\text{max}} = \left[ \frac{P_t G^2 \lambda \sigma}{(4\pi)^3 P_{\text{min}}} \right]^{1/4} \propto \left[ \sigma \right]^{1/4}
  \]

  * \( R_{\text{max}} \) reduced with smaller \( \sigma(RCS) \): shorter radar detection range!
A pulse radar at 10 GHz ($\lambda = 3$ cm $= 0.03$ m)

transmitter power: $P_t = 2$ kW $= 2 \times 10^3$ W  
antenna gain: $G = 28$ dB ($\Rightarrow 631$)

target RCS: $\sigma = 12$ m$^2$

radar minimum detectable signal power: $P_{\text{min}} = -90$ dBm ($\Rightarrow 10^{-12}$ W)

Radar maximum detectable range

$$R_{\text{max}} = \left[ \frac{P_t G^2 \sigma^2}{(4\pi)^3 P_{\text{min}}} \right]^{1/4}$$

$$= \left[ \frac{(2 \times 10^3)(631)^2(0.03)^2(12)}{(4\pi)^3(10^{-12})} \right]^{1/4} = 8.114 \times 10^3 \text{ m}$$

As $R_{\text{max}} \propto \sigma^{1/4}$

If $\sigma$ reduced to $6$ m$^2$ (half size of $12$ m$^2$), $R_{\text{max}} \Rightarrow 8114 \times (0.5)^{1/4} = 6.8 \times 10^3$ m

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Radar Equation & maximum detectable distance

Electromagnetic Communication Lab (EMcom Lab), EE NCKU 2008

**Frequency (MHz):**

$\omega := \pi f$

$\omega := \pi \times 10^6$

$\omega := 2 \pi f$

**Wavelength (m):**

$\lambda := \frac{3 \times 10^8}{f}$

$\lambda = 0.03$

**Transmitter Power (W):**

$P_t := 2000$

$P_{\text{dB}} := 10 \log(P_t)$

$P_{\text{dBW}} = 33.01$

**Antenna gain (dB):**

$G := 10$

$G := 10 \log(P_t)$

$G = 630.957$

**Target RCS (m$^2$):**

$\sigma := 12$

$\sigma := 10 \log(\sigma)$

$\sigma_{\text{dB}} = 10.792$

**Radar min. detectable Power (dBm):**

$P_{\text{min dBm}} := -90$

$P_{\text{min dBm}} := 10$

$P_{\text{min dBm}} := 10^9$

$P_{\text{min dBm}} := 10^{12}$

**Radar max. detectable range (m):**

$$R_{\text{max m}} = \sqrt[4]{\frac{P_t G^2 \sigma^2}{4(4\pi)^3 P_{\text{min W}}}}$$

$$R_{\text{max m}} = 8.113 \times 10^3$$
RCS of a Perfectly Conducting Sphere

A perfectly conducting sphere of radius \( a \) is illuminated by an uniform plane EM wav in \(+z\) direction

\[
\sigma = \frac{\lambda^2}{4\pi(ka)^2} \left| \sum_{n=1}^{\infty} \frac{(-1)^n(2n+1)}{\frac{\partial}{\partial R} R h_n^2(kR)_{R=a}} \right|^2
\]

It can be proven that

- **For a small conducting sphere:** (\( a \ll \lambda \))

\[
\lim_{a \ll \lambda} \sigma \approx \frac{9\lambda^2}{4\pi} (ka)^6 = 144\pi^5 \left(\frac{a^6}{\lambda^4}\right) \propto \lambda^{-4} \quad (\text{wavenumber: } k = 2\pi/\lambda)
\]

\( \Rightarrow \) Rayleigh region \(*\) (accurate for \(a/\lambda < 0.1\))

- **For a large conducting sphere:** (\( a \gg \lambda \))

\[
\lim_{a \gg \lambda} \sigma \approx \pi a^2
\]

\( \Rightarrow \) Optical region \(*\) (accurate for \(2\pi a/\lambda > 10\))

**EX:** For a 60-GHz radar system: \( \lambda = 5 \text{ mm} \) @ \( f = 60 \text{ GHz} \), \( a = 5 \text{ cm} \)

\[
\Rightarrow 2\pi a/\lambda = 62.8 \gg 1
\]

\( \text{RCS: } \sigma \approx \pi a^2 = 7.85 \times 10^{-3} \text{ (m}^2\text{)} \)

If radar antenna gain: \( G = 20 \text{ dB} \), \( R = 1 \text{ m} \)
& \( P_t = 0 \text{ dBm (0 mW)} \)

\[
\Rightarrow P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} = -60 \text{ dBm}
\]
Radar Equation & Radar Cross Section (RCS)

Electromagnetic Communication Lab (EMcom LAB), EE NCKU 2009

Frequency (MHz):
\[ f_{MHz} := 60000 \quad \text{GHz} := \text{GHz} \times 10^6 \quad \omega := 2 \pi f \]

Wavelength (m):
\[ \lambda := \frac{3 \times 10^8}{f} \quad \lambda = 5 \times 10^{-3} \]

Transmitter Power (mW):
\[ P_t \text{mW} := 1 \quad P_t \text{dBm} := 10 \log(P_t \text{mW}) \quad P_t \text{dBm} = 0 \]

Antenna gain (dB):
\[ G_{\text{dB}} := 20 \quad G := 10^{\frac{G_{\text{dB}}}{10}} \quad G = 100 \]

Distance (m):
\[ R := 1 \]
\[ a := 0.05 \quad \left(\frac{2 \pi a}{\lambda}\right) = 62.832 \]
\[ \sigma := \pi a^2 \quad \sigma = 7.854 \times 10^{-3} \left(\text{m}^2\right) \]

Radiation power density at target (W/m²):
\[ S_r = \frac{P_t \text{mW} \times 0.001}{(4 \pi R^2)} \quad S_r = 7.958 \times 10^{-3} \] (mW/cm²):

Back-scattering power density at radar antenna (W/m²):
\[ S_b := \frac{\sigma}{(4 \pi R^2)} \quad S_b = 4.974 \times 10^{-6} \]

Radar antenna receiving power (W):
\[ \text{Pr} := S_r \frac{\lambda^2}{4 \pi} \quad \text{Pr} = 9.895 \times 10^{-10} \quad \text{Pr} := P_t \text{mW} \times 10^{\frac{G}{10}} \quad \text{Pr} := 9.895 \times 10^{-7} \]

or
\[ \text{Pr} := S_b \frac{\lambda^2}{4 \pi} \quad \text{Pr} = 9.895 \times 10^{-10} \quad \text{Pr} := P_t \text{mW} \times 10^{\frac{G}{10}} \quad \text{Pr} := 9.895 \times 10^{-7} \]

* Two-way path loss (dB):
\[ 10 \log \left(\frac{\lambda^2}{4 \pi R^4}\right) = -100.046 \]

**TABLE 12.3 Typical Radar Cross Sections**

<table>
<thead>
<tr>
<th>Target</th>
<th>(\sigma) (m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bird</td>
<td>0.01</td>
</tr>
<tr>
<td>Missile</td>
<td>0.5</td>
</tr>
<tr>
<td>Person</td>
<td>1.0</td>
</tr>
<tr>
<td>Small plane</td>
<td>1–2</td>
</tr>
<tr>
<td>Bicycle</td>
<td>2.0</td>
</tr>
<tr>
<td>Small boat</td>
<td>2.0</td>
</tr>
<tr>
<td>Fighter plane</td>
<td>3–8</td>
</tr>
<tr>
<td>Bomber</td>
<td>30–40</td>
</tr>
<tr>
<td>Large airliner</td>
<td>100</td>
</tr>
<tr>
<td>Truck</td>
<td>200</td>
</tr>
</tbody>
</table>
A pulse radar system & timing diagram

- A pulse radar determines target range by measuring the round-trip time of a pulsed microwave signal.

- The transmitter portion consists of a single-sideband mixer used to frequency offset a microwave oscillator of frequency $f_0$ by an amount equal to the IF frequency.

- After power amplification, pulses of this signal are transmitted by the antenna. The transmit/receive switch is controlled by the pulse generator to give a transmit pulse width $\tau$, with a pulse repetition frequency (PRF) of $fr = 1/Tr$.

- The transmit pulse thus consists of a short burst of a microwave signal at the frequency $f_0 + f_{IF}$.

- Typical pulse durations range from 100 ms to 50 ns; shorter pulses give better range resolution, but longer pulses result in a better SNR after receiver processing.

- Typical pulse repetition frequencies range from 100 Hz to 100 kHz; higher PRFs give more returned pulses per unit time, which improves performance, but lower PRFs avoid range ambiguities that can occur when $R > cTr / 2$. 

FIGURE 14.21  A pulse radar system and timing diagram.
In the receive mode, the returned signal is amplified and mixed with the local oscillator of frequency $f_0$ to produce the desired IF signal.

The local oscillator is used for both up-conversion in the transmitter and down-conversion in the receiver; this simplifies the system and avoids the problem of frequency drift, which would be a consideration if separate oscillators were used.

The IF signal is amplified, detected, and fed to a video amplifier/display. Search radars often use a continuously rotating antenna for $360^\circ$ azimuthal coverage; in this case the display shows a polar plot of target range versus angle. Modern radars use a computer for the processing of the detected signal and display of target information.

The transmit/receive (T/R) switch in the pulse radar actually performs two functions: forming the transmit pulse train, and switching the antenna between the transmitter and receiver.

This latter function is also known as duplexing. In principle, the duplexing function could be achieved with a circulator, but an important requirement is that a high degree of isolation (about 80–100 dB) be provided between the transmitter and receiver to avoid transmitter leakage into the receiver, which would drown the target return (or possibly damage the receiver).

As circulators typically achieve only 20–30 dB of isolation, some type of switch, with high isolation, is required. If necessary, further isolation can be obtained by using additional switches along the path of the transmitter circuit.
Doppler radar system

Doppler effect: \( f_d = \frac{2vf_o}{c} \)

If the target has a velocity component along the line of sight of the radar, the returned signal will be shifted in frequency relative to the transmitted frequency.

- transmitted frequency is \( f_0 \)
- radial target velocity is \( v \)
- received frequency is then \( f_0 \pm f_d \),
  - “+” plus corresponds to an approaching target
  - “-” minus sign corresponds to a receding target

Since the return of a pulse radar from a moving target will contain a Doppler shift, it is possible to determine both the range and velocity (and position, if a narrow-beam antenna is used) of a target with a single radar.

- Such a radar is known as a pulse-Doppler radar
- it offers several advantages over pulse or Doppler radars.
- One problem with a pulse radar is that it is impossible to distinguish between a true target and clutter returns from the ground, trees, buildings, etc.
- Such clutter returns may be picked up from the antenna sidelobes.

However, if the target is moving (e.g., as in an airport surveillance radar application), the Doppler shift can be used to separate its return from clutter.
A multichannel microwave radiometer used to measure the water vapor profile of the atmosphere. This system has one receiver that operates at 36.5 GHz to sense liquid water in the atmosphere, and a second group of receivers operating from 16 to 28 GHz to sample the 22GHz water vapor resonance.

Environmental Applications
- Measurement of soil moisture
- Flood mapping
- Snow cover/ice cover mapping
- Ocean surface windspeed
- Atmospheric temperature profile
- Atmospheric humidity profile

Military Applications
- Target detection
- Target recognition
- Surveillance
- Mapping

Courtesy of the Microwave Remote Sensing Laboratory, University of Massachusetts at Amherst
Total power radiometer block diagram

Balanced Dicke radiometer block diagram
Noncontact millimeter-wave life detection system (MLDS)
For human vital-signs monitoring
(Cardiorespiratory Doppler Radar Sensor)

60-GHz MLDS Prototype Measurements: \( R = 2 \text{ m} \) (Single Mixer)
心電圖
(Eletrocardiogram: ECG)