Appendix 4B  Formulæ for parallel-coupled microstrip transmission lines

These low frequency expressions, as given by Kirschning and Jansen [A4.1] (© 1984, IEEE), make use of normalized values for the strip width and spacing

\[
\frac{u}{h} = \frac{w}{h} \quad \text{and} \quad \frac{g}{h} = \frac{s}{h}
\]

(A4.1)

They are modified here only to the extent that consistent terminology is maintained. A sample set of data with intermediate calculation values is provided in the table following the expressions, to assist in checking any computer implementation of the equations. Two equal width strips of negligible thickness are assumed. The two expressions, \(Z_0\) and \(\varepsilon_{\text{eff}}\), as they appear in this section relate to a single microstrip line of width \(w\) on the same substrate material and are derived from the analysis formulae of Table 3.2. For the following range of parameters:

\[
0.1 \leq u \leq 10.0 \quad 0.1 \leq g \leq 10.0 \quad 1.0 \leq \varepsilon_{r} \leq 18.0
\]

the errors quoted in [A4.1] are \(< 0.7\%\) for \(\varepsilon_{\text{eff}}^{(e)}\), \(< 0.5\%\) for \(\varepsilon_{\text{eff}}^{(o)}\), and \(< 0.6\%\) for \(Z_{0e}\) and \(Z_{0o}\).

\[
\varepsilon_{\text{eff}}^{(e)} = \frac{\varepsilon_{r} + 1}{2} + \frac{\varepsilon_{r} - 1}{2} \left[ 1 + \frac{10}{v} \right]^{-a_{e}(v) \times b_{e}(\varepsilon_{r})}
\]

(A4.2)

with

\[
v = \frac{u (20 + g^2)}{10 + g^2} + g \times \exp(-g)
\]

\[
a_{e}(v) = 1 + \frac{1}{49} \times \ln \left[ \frac{v^4 + (v/52)^2}{v^4 + 0.432} \right] + \frac{1}{18.7} \times \ln \left[ 1 + \left( \frac{v}{18.1} \right)^3 \right]
\]

\[
b_{e}(\varepsilon_{r}) = 0.564 \times \left[ \frac{\varepsilon_{r} - 0.9}{\varepsilon_{r} + 3.0} \right]^{0.053}
\]

THE ODD-MODE EFFECTIVE PERMITTIVITY

\[
\varepsilon_{\text{eff}}^{(o)} = \varepsilon_{\text{eff}} + \left[ \frac{\varepsilon_{r} + 1}{2} + a_{o}(u, \varepsilon_{r}) - \varepsilon_{\text{eff}} \right] \times \exp(-c_{0} \times g_{0}^2)
\]

(A4.3)
with \[ a_0(u, \varepsilon_r) = 0.7287 \times \left( \varepsilon_{\text{eff}} - \frac{\varepsilon_r + 1}{2} \right) \times (1 - \exp(-0.179 u)) \]

\[ b_0(\varepsilon_r) = \frac{0.747 \varepsilon_r}{0.15 + \varepsilon_r} \]

\[ c_0 = b_0(\varepsilon_r) - (b_0(\varepsilon_r) - 0.207) \times \exp(-0.414 u) \]

\[ d_0 = 0.593 + 0.694 \times \exp(-0.562 u) \]

**THE EVEN-MODE CHARACTERISTIC IMPEDANCE**

\[ Z_{0e} = Z_0 \times \left[ \frac{\varepsilon_{\text{eff}}}{\varepsilon(0)} \right]^{\frac{1}{2}} + \left[ 1 - \frac{\varepsilon_{\text{eff}} Z_0 Q_4}{377} \right] \]  

(A4.4)

with

\[ Q_1 = 0.8695 \times u^{0.194} \]

\[ Q_2 = 1 + 0.7519g + 0.189 \times g^{2.31} \]

\[ Q_3 = 0.1975 + \left[ 16.6 + \left( \frac{8.4}{g^6} \right) \right]^{-0.387} + \frac{1}{241} \times \ln \left( \frac{g^{10}}{1 + (g/3.4)^{10}} \right) \]

\[ Q_4 = \frac{2Q_1}{Q_2} \times \left[ u^{Q_3 \times \exp(-g)} + u^{-Q_3(2 - \exp(-g))} \right]^{-1} \]

**THE ODD-MODE CHARACTERISTIC IMPEDANCE**

\[ Z_{0o} = Z_0 \times \left[ \frac{\varepsilon_{\text{eff}}}{\varepsilon(0)} \right]^{\frac{1}{2}} + \left[ 1 - \frac{\varepsilon_{\text{eff}} Z_0 Q_{10}}{377} \right] \]  

(A4.5)

with

\[ Q_5 = 1.794 + 1.14 \times \ln \left( 1 + \frac{0.638}{g + 0.517 \times g^{2.43}} \right) \]

\[ Q_6 = 0.2305 + \frac{1}{281.3} \times \ln \left( \frac{g^{10}}{1 + (g/5.8)^{10}} \right) + \frac{\ln(1 + 0.598 \times g^{1.154})}{5.1} \]

\[ Q_7 = \frac{10 + 190 \times g^2}{1 + 82.3 \times g^3} \]

\[ Q_8 = \exp(-6.5 - 0.95 \times \ln(g) - (g/0.15)^5) \]

\[ Q_9 = \left[ Q_8 + \frac{1}{16.5} \right] \times \ln(Q_7) \]

\[ Q_{10} = Q_4 - \frac{Q_4 - Q_5 \times \exp(\ln(u) \times Q_6 \times u^{-Q_9})}{Q_2} \]
Figure 8.5  The even- and odd-mode impedances for coupled microstrip transmission lines, including the condition $Z_{0e} Z_{0o} = (50)^2$, (dotted curve)

Figure 8.7  Normalized even- and odd-mode effective permittivities for directional coupler design, with the condition $Z_{0e} Z_{0o} = (50)^2$ applying
For directional couplers that are to be matched to the input feed lines, it was seen in (8.15) that only certain combinations of $Z_{0e}$ and $Z_{0o}$ would be required. As the majority of systems are designed for 50Ω characteristic impedance interconnections, the required condition (8.15) is also plotted in Figure 8.5.

Figure 8.6 The line separation and width for a single section of an edge-coupled directional coupler, matched to 50Ω input and output lines
Table A4.1 Sample calculated values to verify the coupled-line equations

<table>
<thead>
<tr>
<th>SAMPLE VALUES</th>
<th>( u = \frac{V}{h} )</th>
<th>( g = \frac{V}{h} )</th>
<th>( \varepsilon_r = 2.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFFECTIVE PERMITTIVITIES</td>
<td>( \varepsilon_{\text{eff}}^{(e)} )</td>
<td>( \varepsilon_{\text{eff}}^{(o)} )</td>
<td>( a_0(u, \varepsilon_r) )</td>
</tr>
<tr>
<td>( \varepsilon_{\text{eff}}^{(e)} )</td>
<td>2.1649</td>
<td></td>
<td>1.8870</td>
</tr>
<tr>
<td>( \nu )</td>
<td>4.8472</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_c(\nu) )</td>
<td>1.0010</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b_0(\varepsilon_r) )</td>
<td>0.5283</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

MODE CHARACTERISTIC IMPEDANCES

| \( Z_{0e} \) | 66.84 | \( Z_{00} \) | 45.10 |
| \( Q_1 \) | 1.0220 | \( Q_5 \) | 2.6237 |
| \( Q_2 \) | 1.4141 | \( Q_6 \) | 0.2525 |
| \( Q_3 \) | 0.1702 | \( Q_7 \) | 0.0000 |
| \( Q_4 \) | 0.7575 | \( Q_8 \) | 0.0000 |

REFERENCE


\[
\begin{align*}
\mathcal{V}^{(e)} &= \frac{C}{\sqrt{\varepsilon_{\text{eff}}^{(e)}}} \\
\mathcal{V}^{(o)} &= \frac{C}{\sqrt{\varepsilon_{\text{eff}}^{(o)}}} \\
\varepsilon_{\text{eff}} &= \left( \frac{\varepsilon_{\text{eff}}^{(e)}}{2} + \frac{\varepsilon_{\text{eff}}^{(o)}}{2} \right)
\end{align*}
\]

Field distributions resulting from (a) even-mode and (b) odd-mode excitation of parallel-coupled microstrip lines